

Inner products on other vector spaces

1. You have two discrete signals (infinite sequences) $x = (1, 0, 1, 0, 1, \dots)$ and another $y = (1, 0.5, 0.25, 0.125, 0.0625, \dots)$. The first represents a switch that is alternatingly opened and closed, and the second is a response to a given shock. What is the inner product between these two vectors?

2. Recall that $e^z e^w = e^{z+w}$ and that $(e^z)^* = e^{(z^*)}$. The integral of a real-valued function is by definition the area under the curve, so if we consider functions defined on the interval $[-5 \times 10^{-9}, 5 \times 10^{-9}]$ (suppose we have two large parallel metal plates separated by 10 nm, so one is at -5 nm and the other is at 5 nm in a direction perpendicular to the plates). Thus, the inner product between two wave functions is defined as

$$\langle f, g \rangle = \int_{-5 \times 10^{-9}}^{5 \times 10^{-9}} f^*(x) g(x) dx. \text{ If } f(x) = e^{j\omega x} \text{ where } \omega > 0 \text{ is a constant, what is } \langle f, f \rangle?$$

Hint, even though you have not yet learned integration, there really is enough information here to get the right answer. For example, recall from elementary school that the area of a rectangle is base times height.

3. Normalize the vector f in Question 2.

Note: f is a function, and $f(x)$ is that function evaluated at x , so $\sin(t)$ is a real value, while $\sin$ is a function that oscillates as time progresses. This is the same as $\mathbf{u}$ being a vector and u_k being an entry in that vector, or x being an infinite sequence and $x[10]$ being the eleventh entry in that sequence (recalling that the first entry is $x[0]$).